Fuzzy Knowledge Based Controllers Verification

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Abstract

The application of the fuzzy logic to the control theory by means of fuzzy knowledge based controllers (FKBC) has improved the results, especially in dynamic systems or systems subject to great disturbances. One of the essential elements of an FKBC is the fuzzy rule base that represents the expert knowledge. This must fulfill certain structural demands which assure its correctness. In the present article a mechanism is proposed in order to detect possible contradictions in the base by the association of a certain degree of contradiction to each pair of rules, according to an analysis of the similarity between their antecedents and consequents. Finally, an example is used to illustrate this method.

1 Introduction

In general, software engineering has always been based on the obtained solution with a series of requirements or properties which would enable us to consider its validity and correctness. This task -essential to establish the system’s credibility and reliability and, therefore, its usefulness as a convenient tool- is becoming more and more important as the designed systems increase their complexity. Consequently, the effectiveness of the traditional methods on software verification and validation has decreased.

Knowledge Based Systems [2] (KBS) are a type of software which, due to their characteristics, make the validation and verification tasks a crucial and, at the same time, extremely complicated stage. Crucial because, in many cases, it implies important economic and social consequences that could even have an effect on our own life (as in the case of medical diagnostic systems). Complicated since it involves the treatment of knowledge which, by its nature, usually implies great amount of subjectivity and imprecision, preventing the experts from establishing general and objective paths in order to check the correct extraction, exposition and exploitation of its knowledge.

KBS knowledge may contain several structural anomalies which make it inconsistent or incomplete, such as redundancy, circularity, deficiency or ambivalence [4][5][6][8]. KBS verification must be able to detect those anomalies and provide the necessary facilities to enable the knowledge engineer to correct them.

In fact, KBS credibility and acceptance from the final user will depend to a large extent on the results of the verification and validation stages, as well as on how exhaustively they are carried out. If a user does not believe in the answers of a system, it will not be useful anymore. So its credibility is closely related to the capability of assuring the system’s correctness.

Furthermore, KBS verification and validation tasks can provide the expert with information on his knowledge, on aspects such as its structure or the way he uses it.

2 Fuzzy Controllers

The control is a discipline that tries to rule the running of a system analyzing information about its state, which it will know by some state or input variables, and acting on it through one or more control or output variables which will modify that state, creating a cyclical flow of information and processes (figure 1).

Traditionally, controllers design has been approached from the viewpoint of conventional control engineering, producing different types of analytical solutions. Nevertheless, these techniques have proved not to be very flexible in many cases, decreasing its efficiency in dynamic systems or in those subject to great distur-
bances.

The KBS have allowed us to improve the running of analytical models by means of the incorporation of a kind of knowledge hardly integrable in them, giving rise to producing the so-called Knowledge Based Controllers (KBC). Within this group there are the Fuzzy Knowledge Based Controllers [1][9] (FKBC), whose knowledge base consists of a set of rules where state and control variables are represented by linguistic variables [11]. Each of these variables takes its values from a set of fuzzy labels defined on the domain of the associated variable of the system.

Every fuzzy rule is composed of an antecedent formed by a combination of fuzzy restrictions on the state variables and a consequent that consists of assigning a fuzzy restriction to the control variable. That is to say, every fuzzy rule has this form:

\[ R^{(i)}: \text{if } X_1 \text{ is } A_1^{(i)} \text{ and...and } X_n \text{ is } A_n^{(i)} \text{ then } Y \text{ is } B^{(i)} \]

where \( X_j \) are the state variables, \( Y \) is the control variable, and \( A_j^{(i)} \) and \( B^{(i)} \) are the fuzzy labels on the domains \( U_j \) and \( V \), respectively.

Moreover, a mechanism of inference is needed to fire the fuzzy rules according to the state of the system (or, in other words, the value of the state variables), generating an action (i.e., an assignment to the control variable).

Given a fuzzy rule

\[ R: \text{if } X \text{ is } A \text{ then } Y \text{ is } B \]

the first step to infer something from it will be to give the rule a meaning so that a relation between the antecedent and the consequent could be established. In control, the most used method is Mamdani’s implication [3].

Next it will be necessary to determine the mechanism of inference itself in such a way that, for a value of \( X \) equals \( A' \), the value induced by \( R \) for \( Y \) would be the function of \( R \) meaning and \( A' \). For this purpose, a combination of the maximum and minimum operators is normally used, and is called sup-min composition.

In short, when fuzzy control is considered, where the \( u^* \) values that state variables assume are crisp values, the inference using Mamdani’s implication and the sup-min composition provides a value for \( Y \) which is a version of \( B \) cut at the height of \( \mu_A(u^*) \), i.e.,

\[ B^* = \int_y \min(\mu_A(u^*), \mu_B(v_j))/v_j \]

showed by the picture on figure 2.

![Figure 2: Sup-min composition of the crisp value \( u^* \) and the rule \( R \) expressed by Mamdani’s implication.](image)

### 3 Fuzzy Controllers Verification

The knowledge immersed in the rule base of a FKBC can contain some of the structural anomalies likely to find in any KBS, frequently produced by the incorrect translation of the knowledge from the expert (in case he exists) or the training base (in case of the machine learning) to the system.

Among them, we can distinguish the contradictions due to their importance in the control field.

In classical logic, it is said that two rules are contradictory if their antecedents are identical and their consequents are opposite. For instance, the following pair of rules is contradictory

\[ \begin{align*}
  & R^1: \text{if } A \text{ and } B \text{ then } Z \\
  & R^2: \text{if } A \text{ and } \neg B \text{ then } \neg Z
\end{align*} \]

In a fuzzy rule base, determining whether two propositions are contradictory or not is more complicated, because of the uncertainty subjacent to the fuzzy logic. Normally, a pair of fuzzy rules has been considered contradictory if their antecedents are identical and their consequents disjoint, that is to say, if the fuzzy values associated to the output variable do not intersect in any of the domain points. For example, if the domain associated to the input and output variables of a fuzzy rule base was that of the figure 3, the following rules \( R^{(1)} \) and \( R^{(2)} \) would be contradictory:

\[ \begin{align*}
  & R^{(1)}: \text{if } X_1 \text{ is } HN \text{ and } X_2 \text{ is } SN \text{ then } Y \text{ is } SN \\
  & R^{(2)}: \text{if } X_1 \text{ is } HN \text{ and } X_2 \text{ is } SN \text{ then } Y \text{ is } SP
\end{align*} \]
This valuation of the contradiction between a pair of rules in absolute values of truth seems to be in conflict with the imprecision the use of fuzzy rules tries to reflect on the system. Therefore, it seems more appropriate to measure the contradiction level of a pair of rules examining them in depth.

Intuitively, given two rules with identical antecedents, if their consequents are also identical, the degree of contradiction presented will be minimum, whereas it will be maximum if their consequents are opposed. The rest of the situations, which will be distributed between these two extremes, will have to affect the degree of contradiction of the pair of rules proportionally to the degree of dissimilarity between the two consequents. For example, the following rule \( R^{(3)} \) should obtain a degree of contradiction in relation to \( R^{(1)} \) inferior to \( R^{(2)} \), but big enough to reflect the incompatibility between both output answers:

\[ R^{(3)} : \text{if } X_1 \text{ is } HN \text{ and } X_2 \text{ is } SN \text{ then } Y \text{ is } Z \]

An approach to the above exposition is to consider the degree of contradiction of a pair of rules \((R^{(i)}, R^{(j)})\) with identical antecedents equal to a degree of dissimilarity between their consequents \(B^{(i)}\) and \(B^{(j)}\). This degree can be calculated in terms of the mean distance between the values of both consequents [7], more specifically like

\[
d(B^{(i)}, B^{(j)}) = \frac{\sum \sum_{v_1, v_2 \in V} [v_1 - v_2] \inf[\mu_{B^{(i)}}(v_1), \mu_{B^{(j)}}(v_2)]}{\sum \sum_{v_1, v_2 \in V} \inf[\mu_{B^{(i)}}(v_1), \mu_{B^{(j)}}(v_2)]}
\]

An objection to this measure is that it is not normalized. To overcome it, the maximum mean distance can be obtained in order to use the following new formula:

\[
d_N(B^{(i)}, B^{(j)}) = \frac{d(B^{(i)}, B^{(j)})}{d_{\text{max}}}
\]

To carry out the calculations it is necessary to represent the domain of the figure 3 in tabular form (table 3). In it, the maximum mean distance between two fuzzy label is:

\[d_{\text{max}} = d(HN, HP) = 24.00\]

and, therefore, the degree of contradiction between every two of the rules \( R^{(1)}, R^{(2)} \) and \( R^{(3)} \) will be:

\[
d_N(B^{(1)}, B^{(2)}) = d_N(SN, SP) = 0.33
\]
\[
d_N(B^{(1)}, B^{(3)}) = d_N(SN, Z) = 0.17
\]
\[
d_N(B^{(2)}, B^{(3)}) = d_N(SP, Z) = 0.17
\]

Moreover, keeping our position about measuring the contradiction state between pairs of rules, what would it happen if the antecedents were not identical, but very similar? Should it be considered that a contradiction exists? Again, the solution seems to depend on the degree of similarity or proximity between the antecedents. Now, the degree of proximity of the antecedents will act upon what has been considered so far the degree of contradiction of the rule; in other words, upon the degree of dissimilarity between the consequents. Thus, the rule \( R^{(3)} \) of the form

\[ R^{(3)} : \text{if } X_1 \text{ is } HN \text{ and } X_2 \text{ is } MN \text{ then } Y \text{ is } Z \]

should have a degree of contradiction in relation to \( R^{(1)} \) inferior to \( R^{(3)} \), since the antecedents of the latter are more similar.

In order to reach our aim firstly it will be necessary to determine the method to calculate the degree of similarity between two conjunctions of fuzzy propositions. As a measure, we will use the arithmetic mean of the
opposite the partial mean distances between each pair of propositions; that is to say, given two premises:

\[ P(i) : X_1 = A_{i1} \] and ... and \( X_n = A_{in} \)

\[ P(j) : X_1 = A_{j1} \] and ... and \( X_n = A_{jn} \)

the degree of similarity between them will be

\[ s_N(P(i), P(j)) = 1 - \frac{\sum_{k=1}^{n} d_N(A_{ik}, A_{jkl})}{n} \]

Lastly, the final degree of contradiction will be

\[ C(R^{(i)}, R^{(j)}) = d_N(B^{(i)}, B^{(j)})t-norm s_N(P^{(i)}, P^{(j)}) \]

The product has been chosen as \( t\)-norm, because it allows to combine in the final result both the degree of dissimilarity of the consequent and the degree of similarity of the antecedent. This would not occur, for instance, with the minimum operator.

Therefore, in our example the degree of contradiction between the rules \( R^{(1)} \) and \( R^{(3)} \) will be:

\[ R^{(1)} : \text{if } X_1 = HN \text{ and } X_2 = SN \text{ then } Y = SN \]

\[ R^{(3)} : \text{if } X_1 = HN \text{ and } X_2 = MN \text{ then } Y = Z \]

\[ d_N(B^{(1)}, B^{(3)}) = d_N(SN, Z) = 0.17 \]

\[ s_N(P^{(1)}, P^{(3)}) = 1 - \frac{d_N(HN, HN) + d_N(SN, MN)}{2} \]

\[ = 1 - \frac{0.08 + 0.17}{2} = 0.875 \]

\[ C(R^{(1)}, R^{(3)}) = d_N(B^{(1)}, B^{(3)}) \times s_N(P^{(1)}, P^{(3)}) = 0.17 \times 0.875 = 0.15 \]

### 4 An Example

In this section we will illustrate the verification method described above by applying it to a practical case. For this purpose we will take the fuzzy rule base used by Yamakawa [10] to control the stabilization of an inverted pendulum. This rule base is formed by 7 rules (table 4), each of which includes in its antecedent two state variables \( \theta \) and \( \dot{\theta} \) that measure, respectively, the angle and the change of angle to the vertical of the pendulum. The consequent will consist of the assignment of a fuzzy value to the control variable \( \ddot{y} \) which represents the speed of the vehicle that moves the pendulum. Both the state and the control variables take their values from a domain formed by the seven fuzzy labels on figure 4.

In order to calculate the degree of contradiction between each pair of rules, a tabular representation of each domain has been considered, discretizing the universe of discourse in which the fuzzy values are defined. For example, the discretization of the fuzzy value \( Z \), from which the rest can be deduced, would be the following:

\[ \{0.07/−1.3,0.14/−1.2,0.21/−1.1,0.29/−1.0,0.36/−0.9,0.43/−0.8,0.50/−0.7,0.57/−0.6,0.64/−0.5,0.71/−0.4,0.79/−0.3,0.86/−0.2,0.93/−0.1,1.00/0.0,0.93/0.1,0.86/0.2,0.79/0.3,0.71/0.4,0.64/0.5,0.57/0.6,0.50/0.7,0.43/0.8,0.36/0.9,0.29/1.0,0.21/1.1,0.14/1.2,0.07/1.3\} \]

The degree of contradiction between each pair of rules is shown in table 3. We can observe that the maximum degree of contradiction appears in the pair of rules \((R^{(3)}, R^{(5)})\), due to the large difference that exists between both consequents, in spite of the fact that only one of the state variables changes its value in the antecedents. The pair of rules \((R^{(1)}, R^{(5)})\) and \((R^{(3)}, R^{(7)})\) also have a high degree of contradiction according to the mean. This could make us consider an alternative configuration so that those degrees of contradiction decrease.

However, the degrees of contradiction obtained should not be interpreted as indicators of the existence of errors in the rule base, but as a system oriented towards possible improvements of that base. These improvements could eliminate or reduce the deficiencies caused

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**Table 2: Rule base of the fuzzy controller**

| \( R^{(1)} \) | if \( \theta \) is \( SN \) and \( \dot{\theta} \) is \( SN \) then \( \ddot{y} \) is \( SN \) |
| \( R^{(2)} \) | if \( \theta \) is \( SP \) and \( \dot{\theta} \) is \( SN \) then \( \ddot{y} \) is \( Z \) |
| \( R^{(3)} \) | if \( \theta \) is \( MN \) and \( \dot{\theta} \) is \( Z \) then \( \ddot{y} \) is \( MN \) |
| \( R^{(4)} \) | if \( \theta \) is \( Z \) and \( \dot{\theta} \) is \( Z \) then \( \ddot{y} \) is \( Z \) |
| \( R^{(5)} \) | if \( \theta \) is \( MP \) and \( \dot{\theta} \) is \( Z \) then \( \ddot{y} \) is \( MP \) |
| \( R^{(6)} \) | if \( \theta \) is \( SN \) and \( \dot{\theta} \) is \( SP \) then \( \ddot{y} \) is \( Z \) |
| \( R^{(7)} \) | if \( \theta \) is \( SP \) and \( \dot{\theta} \) is \( SP \) then \( \ddot{y} \) is \( SP \) |

**Figure 4: Fuzzy domain of the controller.**
Table 3: Degrees of contradiction between pairs of rules

<table>
<thead>
<tr>
<th></th>
<th>$R^{(1)}$</th>
<th>$R^{(2)}$</th>
<th>$R^{(3)}$</th>
<th>$R^{(4)}$</th>
<th>$R^{(5)}$</th>
<th>$R^{(6)}$</th>
<th>$R^{(7)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^{(1)}$</td>
<td>0.065</td>
<td>0.136</td>
<td>0.135</td>
<td>0.142</td>
<td><strong>0.332</strong></td>
<td>0.136</td>
<td>0.046</td>
</tr>
<tr>
<td>$R^{(2)}$</td>
<td>0.065</td>
<td>0.219</td>
<td>0.058</td>
<td>0.275</td>
<td>0.046</td>
<td>0.136</td>
<td>0.046</td>
</tr>
<tr>
<td>$R^{(3)}$</td>
<td>0.065</td>
<td>0.264</td>
<td><strong>0.419</strong></td>
<td>0.275</td>
<td><strong>0.332</strong></td>
<td>0.136</td>
<td>0.046</td>
</tr>
<tr>
<td>$R^{(4)}$</td>
<td>0.065</td>
<td>0.264</td>
<td>0.058</td>
<td>0.142</td>
<td>0.046</td>
<td>0.136</td>
<td>0.046</td>
</tr>
<tr>
<td>$R^{(5)}$</td>
<td>0.065</td>
<td>0.219</td>
<td>0.153</td>
<td>0.046</td>
<td>0.136</td>
<td>0.046</td>
<td>0.046</td>
</tr>
<tr>
<td>$R^{(6)}$</td>
<td>0.065</td>
<td>0.136</td>
<td>0.046</td>
<td>0.136</td>
<td>0.046</td>
<td>0.136</td>
<td>0.046</td>
</tr>
</tbody>
</table>

by the unintentional inclusion of contradictory knowledge.

5 Conclusion

The knowledge base verification of a fuzzy controller is a crucial and, at the same time, extremely complicated stage. In this paper, a solution is presented to determine the degree of contradiction between the rules of this knowledge base. In order to do that, the classic definition of contradictory rules (identical antecedents and opposite consequents) is extended by taking into account the degree of similarity between antecedents and the degree of dissimilarity between consequents. This way, a gradual hint is integrated in the definition, bringing it near the imprecise character of fuzzy controllers.

The degrees of contradiction obtained should be interpreted as a measurement oriented towards modifications in the rule base in order to eliminate or mitigate the undesired effects that could be caused by including contradictory knowledge involuntarily, and not as the reflection of the unequivocal existence of errors in that base.

Therefore, a high degree of contradiction between two rules should alert experts about the apparent inconsistency in the represented knowledge. They must consider whether modifications reducing that degree of contradiction are not incompatible with the behavior expected for the controller. In this sense, experts obtain, by means of the degree of contradiction information about the direction to which they should look for possible improvements of the knowledge base.

References


